

# Hierarchicality of Trade Flow Networks Reveals Complexity of Products

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## Abstract

With globalization, countries are more connected than before by trading flows, which currently amount to at least 36 trillion dollars. Interestingly, approximately 30-60 percent of global exports consist of intermediate products. Therefore, the trade flow network of a particular product with high added values can be regarded as a value chain. The problem is whether we can discriminate between these products based on their unique flow network structure. This paper applies the flow analysis method developed in ecology to 638 trading flow networks of different products. We claim that the allometric scaling exponent  $\eta$  can be used to characterize the degree of hierarchicality of a flow network, i.e., whether the trading products flow on long hierarchical chains. Then, the flow networks of products with higher added values and complexity, such as machinery&transport equipment with larger exponents, are highlighted. These higher values indicate that their trade flow networks are more hierarchical. As a result, without extra data such as global input-output table, we can identify the product categories with higher complexity and the relative importance of a country in the global value chain solely by the trading network.

## Introduction

As the process of globalization accelerates, countries throughout the world are more connected, and collaboration is proceeding in an unprecedented manner under the background of integrated global markets of capital, labor force and products. Consequently, some cross-border production chains, which comprise several countries or regions, have inevitably emerged as the result of international labor force division and collaboration at the global level [1–3]. However, because of the heterogeneities of products, the production networks are very inhomogeneous. Some products in the electronics and automotive industries, such as PCs or automobiles, can be broken down into several independent components and easily transported and assembled in different countries [1]. Therefore, a large fraction of imports for these products are not for final consumption but, rather, are for re-production with higher value-added and exports [1, 4, 5]. Conversely, the networks for agriculture or raw material products may have much shorter production chains. Thereafter the major imports of these products are for final consumption.

Differentiating these products according to the length of their production chains and the level of added-values is of importance for countries' long-term development strategies. The conventional method [6–8] attempts to build the value flow networks among different products directly by incorporating international input-output tables [9–11]. Although the whole picture of production networks can be captured in detail, obtaining accurate raw data on the global level is not easy [8, 10]. However, the highly detailed international trade flow data for various products among countries are well documented with a long history [12, 13]. In particular, all bilateral trade flows are classified by different products according to the SITC (Standard International Trade Coding) or other equivalent coding methods. Therefore, a unique flow structure of one product category can be extracted from the international trade data.

In recent years, the world wide trade network as a specific instance of a complex network has been studied [14–17]. Although both the common features shared by various complex networks and some unique patterns are found, very little attention has been paid to multi-networks of different products [18].

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<sup>1</sup>This paper only represents the personal views of the author

In this paper, we attempt to discriminate products on their level of complexity and value-added by identifying their unique trade flow structures. This is possible because trade networks contain information on global production networks. Almost all of the cross-border product flows in the global value chain are recorded in the international trade data.

Our methodology is to compare the allometric scaling exponents among the flow networks of different products [19, 20]. The allometric scaling pattern has been found to be ubiquitous for trees spanned by binary networks [20, 21], such as food webs [21, 22], trade webs [23] and biological networks [24]. Our previous work has incorporated the flow analysis methods developed in ecology to reveal the common nature of the flow networks in general [22, 25]. It is natural to extend this method to trade flows in which the allometric scaling exponent is given a new explanation, the degree of hierarchicality. This measure characterizes whether the product flows along a long hierarchical chain. We calculate the allometric exponent of each flow network in different product classifications and find that the manufactured products with higher added values have larger exponents. Furthermore, most exponents are larger than one, indicating that the networks are hierarchical, whereas the networks of the primary products with relative low added values have smaller exponents, and the networks are flat. Hierarchicality always indicates inequality and monopoly. We further calculate the relative importance of each country in a product trading network and compare the heterogeneities of the country's impact distribution for different products using the GINI coefficient of country's impact. Finally, the dynamics of allometric scaling exponents along time are shown, and the globalization process can be interpreted.

## Results

### Trade Flow Networks

We use two datasets for study and comparison to eliminate the potential discrepancies in the data. The first one is from Feenstra et al's "World Trade Flows: 1962-2000" dataset based on the United Nations COMTRADE database (abbreviated as the UN dataset) [12]. This dataset covers the bilateral trade flows of approximately 800 types of products according to the SITC 4 (Standard International Trade Classification system, Rev.4) classification standard from 1963 to 2000. In addition, mainly the results from the year 2000 are shown and discussed in the main text. Another dataset (the OECD dataset) is the bilateral trade data from 2009 that was compiled by the Organization of Economic Co-operation and Development (OECD) [13]. The OECD dataset contains only the OECD member countries, so the total number of countries is smaller than the UN dataset. However, these countries dominate approximately 70-80% of the trade volume in the world. The products classification standard of the OECD dataset is ISIC Rev. 3 (International Standard Industrial Classification of All Economic Activities, Rev. 3), which is different from the SITC 4 classification. Please see the detailed discussions of the datasets in the Supporting Information, SI.

The SITC4 codes are hierarchical, meaning that the categories with longer codes are sub-categories of the ones with shorter codes if they share the same prefix. For example, the product category 7 in SITC4 is the category of machinery and transport equipment, so this is a very generalized classification, whereas 71 and 72 are two sub-categories of 7, representing the power machinery product and vehicle categories, respectively.

### Allometric Scaling of Trade Networks

For each product trade network, we can define an exponent  $\eta$  to characterize the hierarchicality of the flow network. First, we need to calculate two vertex-specific variables, namely,  $T_i$  and  $C_i$ .

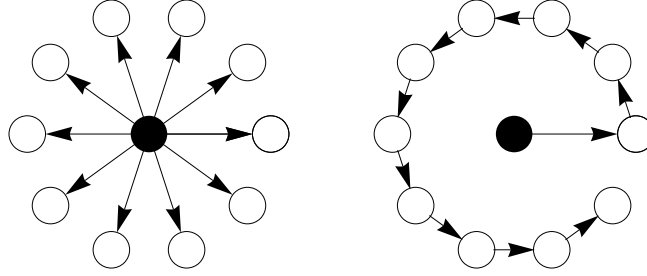
$T_i$ , the trading volume of country  $i$ , is defined as the maximum of  $i$ 's total imports or exports. This reflects the capacity of trade flows through  $i$ . Next,  $C_i$  is the impact of  $i$  on the entire network. It

is defined as the total changes of trading volume of other nodes on the network after the hypothetical deletion of  $i$ . The concrete calculation of these two variables are referred to the method section and supporting information.

Typically, for various empirical trade networks,  $C_i$  and  $T_i$  have a strong correlation which can be described by a power law:

$$C_i \sim T_i^\eta, \quad (1)$$

where  $\eta$  is the allometric scaling exponent. This equation is extended from the empirical allometries from river basins, vascular networks and food webs [20, 21]. Previous studies on spanning trees indicate that the exponent  $\eta$  can be used to reflect the hierarchicality or flatness of a tree. For example, two extreme cases of spanning trees are shown in Figure 1. The star network that has the smallest exponent, 1, is the flattest tree, whereas the chain network that has the largest exponent, 2, is the most hierarchical tree.



**Figure 1.** Two special spanning trees with the minimum allometric exponent 1 (left, a star network) and maximum exponent 2 (right, a directed chain)

This calculation can be extended to general flow networks [22, 25]. Nevertheless the exponent is not bound in  $[1, 2]$ . However, we can also define the exponent as the hierarchicality of a general flow network, as it will contain long flow chains if its exponent is larger (see Supplementary Information).

It turns out that the allometric scaling pattern (Equation 1) is very general for all of the studied trade networks but their exponents are not similar. Figure 2 shows the allometric scaling patterns of two products.

In Figure 2, each data point stands for a country participating in the international trade of this product. The pairs of  $T_i$  and  $C_i$  form a straight line on the log-log coordinate, which means a power law relationship between the two variables exists (i.e., Equation 1). The exponents for these two products are distinct, indicating that the power-generating trade network is more hierarchical than the fruit and vegetable networks. In other words, the production for power generating machines is along a longer value-added chain than fruits and vegetables.

This point can be visualized by the network plots of these two products (Figure 3). Although only the backbone links are shown and other links are faded in the background, it is clear that the upper network has many long chains that always root from some major exporters of power generating machines (e.g. the US and Japan). However, the lower network is more fragmented. Although several large countries (e.g. the US) still occupy a large fraction of the fruit trade, most of them are importers. This implies the whole network lacks a center and is more flat. Intuitively, that is the reason why the exponent of the first network is larger than the latter.

## Exponents Comparison and Distributions

We further compare the exponents among different networks of products in a more systematic way. In Tables 1 and 2, we list exponents for all 1-digit products in the UN dataset and OECD dataset to compare.

**Table 1.** Exponents of 1-digit SITC4 categories in the UN dataset

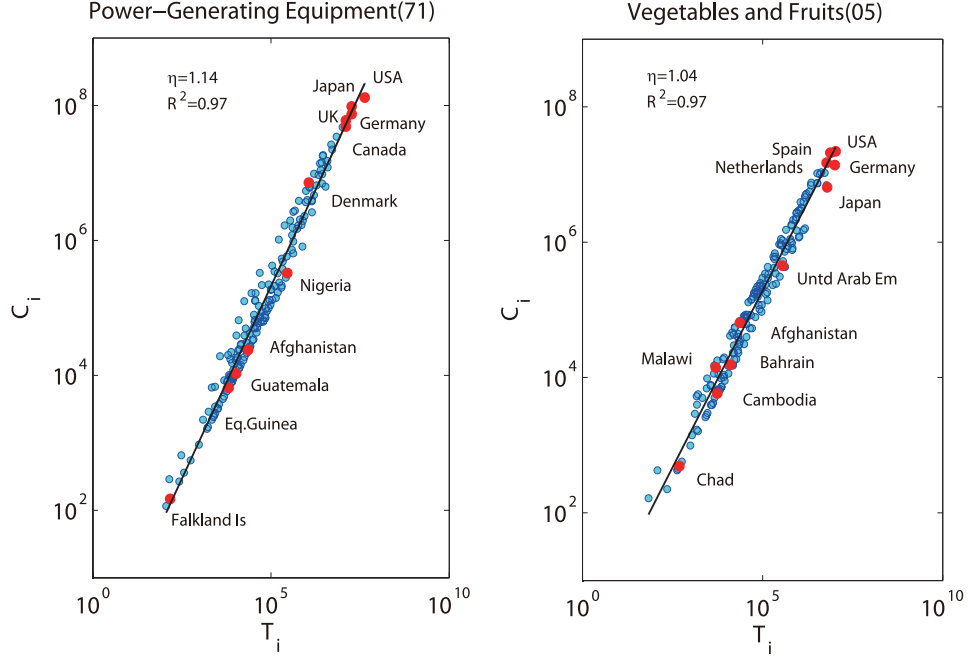
(The categories of 8 (Miscellaneous) and 9(Not classified) are ignored in this table, the last row shows the allometry of all products as an integrated network.)

Code	Classification	$\eta$	$R^2$	GINI
7	Machinery and transport equipment	$1.136 \pm 0.026$	0.974	0.889
6	Manufactured goods classified chiefly by materials	$1.120 \pm 0.026$	0.962	0.830
5	Chemicals and related products	$1.117 \pm 0.034$	0.972	0.877
1	Beverages and tobacco	$1.116 \pm 0.033$	0.958	0.868
4	Animal and vegetable oils, fats and waxes	$1.077 \pm 0.029$	0.973	0.847
0	Food and live animals	$1.043 \pm 0.032$	0.971	0.798
3	Mineral fuels, lubricants and related materials	$1.042 \pm 0.018$	0.954	0.821
2	Crude materials, inedible, except fuels	$1.001 \pm 0.020$	0.988	0.815
-	All Products	$1.022 \pm 0.030$	0.965	0.817

**Table 2.** Exponents for different products in the OECD dataset

(The products in different industries coded by the ISIC Rev. 3 coding system for industries is shown. The financial intermediation, business services, wholesale and retail trade, transport and storage, post and telecommunication, hotels and restaurants, and construction industries are ignored because their trades do not stand for goods flows. The last row shows the allometry of all industries as an integrated network)

Code	Classification	$\eta$	$R^2$	GINI
29	Machinery and equipment, nec	$1.146 \pm 0.072$	0.947	0.656
23T26	Chemicals and non-metallic mineral products	$1.129 \pm 0.079$	0.937	0.563
34T35	Transport equipment	$1.124 \pm 0.075$	0.941	0.669
30T33	Electrical and optical equipment	$1.112 \pm 0.070$	0.948	0.667
27T28	Basic metals and fabricated metal products	$1.092 \pm 0.080$	0.974	0.568
40T41	Electricity, gas and water supply	$1.075 \pm 0.054$	0.931	0.649
36T37	Manufacturing nec; recycling	$1.074 \pm 0.078$	0.967	0.684
15T16	Food products, beverages and tobacco	$1.073 \pm 0.081$	0.931	0.553
20T22	Wood, paper, paper products, printing and publishing	$1.051 \pm 0.088$	0.926	0.589
10T14	Mining and quarrying	$1.019 \pm 0.041$	0.911	0.721
01T05	Agriculture, hunting, forestry and fishing	$1.019 \pm 0.070$	0.978	0.576
17T19	Textiles, textile products, leather and footwear	$0.998 \pm 0.065$	0.949	0.705
-	All industries	$0.941 \pm 0.072$	0.924	0.474



**Figure 2.** The allometric scaling between  $T_i$  (in U.S. dollars) and  $C_i$  (in U.S. dollars) of two networks. The left figure shows a super-linear scaling law (with an exponent larger than 1) for power generating products, whereas the right figure shows a nearly linear scaling law (with an exponent close to 1) for fruits and vegetables

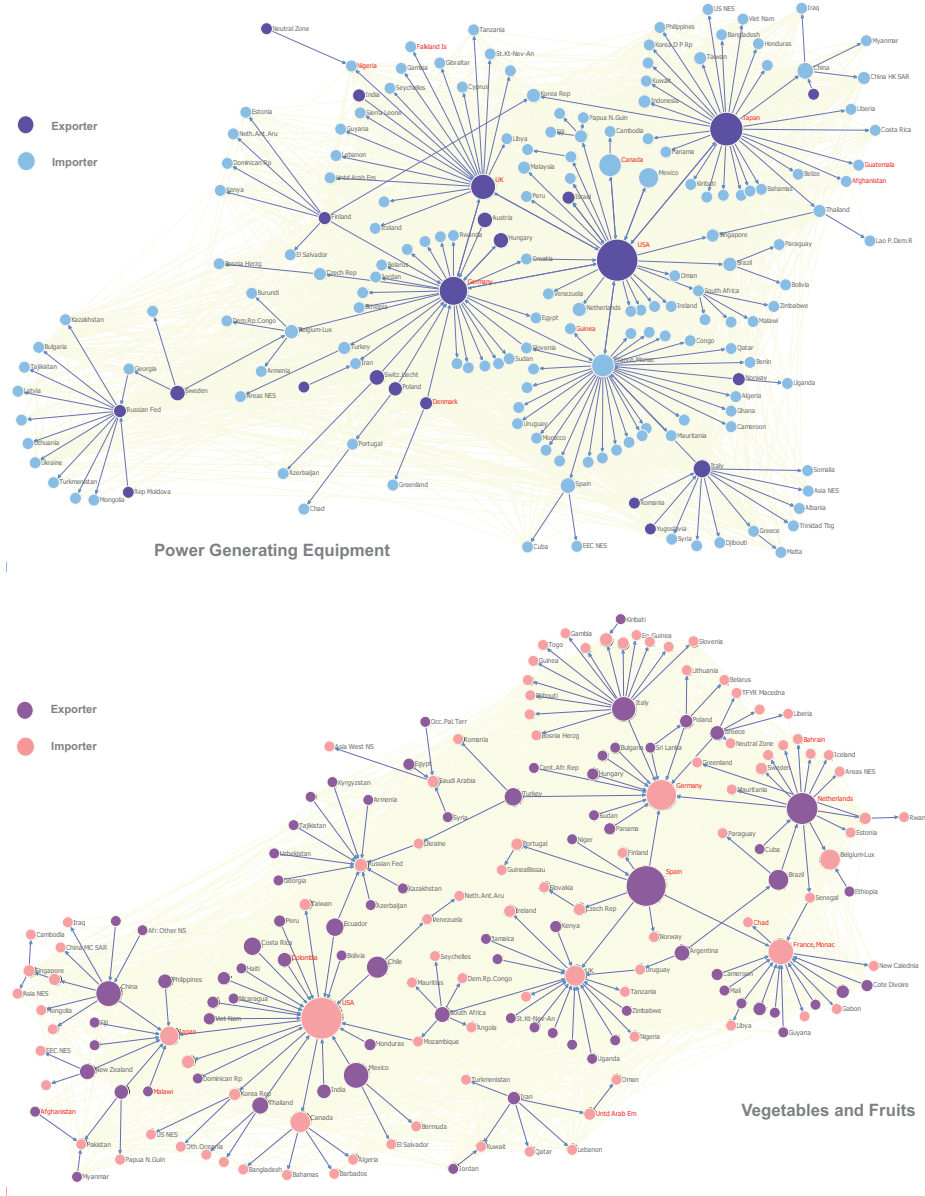
Both tables display large gaps of exponents for different products ( $[1.001, 1.136]$  for the UN dataset and  $[0.944, 1.146]$  for the OECD dataset). Although some slight differences between SITC4 classification and ISIC Rev. 3 classifications exist, the products of machinery, equipment, chemicals and similar are of higher exponents than the products of food, mining and agriculture industries. This unique observation can be further confirmed and extended to finer classifications.

Figure 4 shows the exponent distribution of all products with 4-digit classification in the UN dataset. The frequency curve has a bell-shape peaked at 1.09, which means most product networks are hierarchical. The stacked color bars show the distributions of all 1-digit classifications (Figure 4 left). Note that most blue bars are located in the right side of the bell-shaped curve, whereas the green and yellow bars are located on the left side, indicating that the machinery and manufactured products have larger exponents than the food and beverage products. This phenomenon is better illustrated in the right subplot of Figure 4, in which we simply classify the products as primary products (prefix of 0,1,2,3, and 4) and manufactured products (prefix of 5,6,7,8, and 9).

## Allometric Exponent and Product Complexity

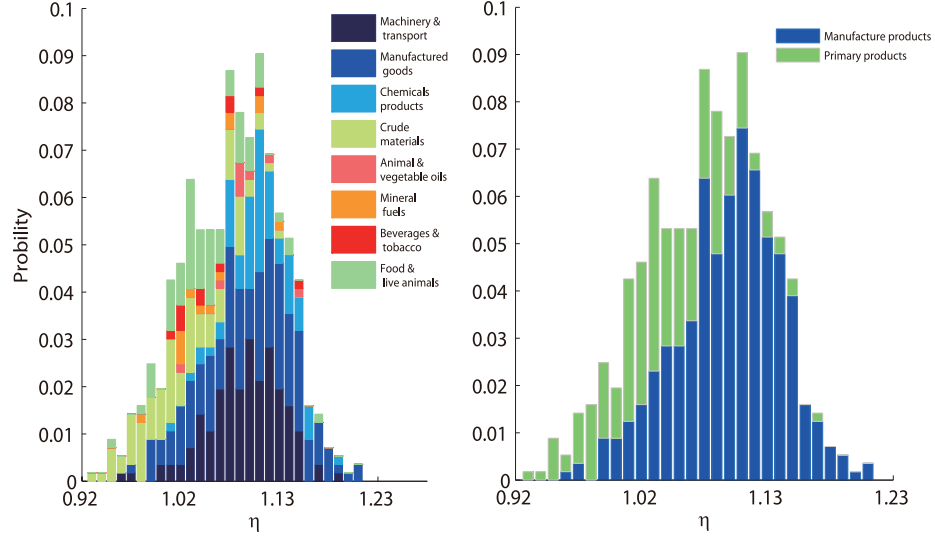
According to the observations, we know that the allometric exponents of the trade flow network can reflect the basic properties of products. The manufactured products with higher added-value and complex production process always have larger exponents. Therefore, we conjecture that a positive correlation between the exponents and the nature of products (complexity or value added) may exist.

To test our hypothesis, we perform two correlation analysis on both datasets. For the UN dataset, we



**Figure 3.** Visualization of trade flow network for power generating equipment (upper) and fruits and vegetables (lower). We use different colors to distinguish nodes as importer (import is larger than its export) and exporter (export is larger than import). The size of node denotes the total volume of trade. In these two networks, only the backbones are shown as the main parts and all other un-important links are hidden as backgrounds. The backbone extracting method is according to [26]

correlate the exponents with PRODY, one of the measurements of product complexity. It is calculated as the average comparative advantage weighted by the GDP per capita of the exporters of this product [27].



**Figure 4.** Exponent Distribution for All of 4-digit SITC4 Product Categories. The stacked bar charts of different colors correspond to 1-digit SITC4 categories (left) and primary and manufacture classifications (right). For one specific 1-digit classification (say 0 for food and living animals), we can calculate the frequencies on each exponent intervals for all products with 0 prefix, then these frequencies as little bars are stacked on the tops of existing bars.

It is calculated as follows:

$$PRODY(p) = \sum_c Y_c RCA(c, p), \quad (2)$$

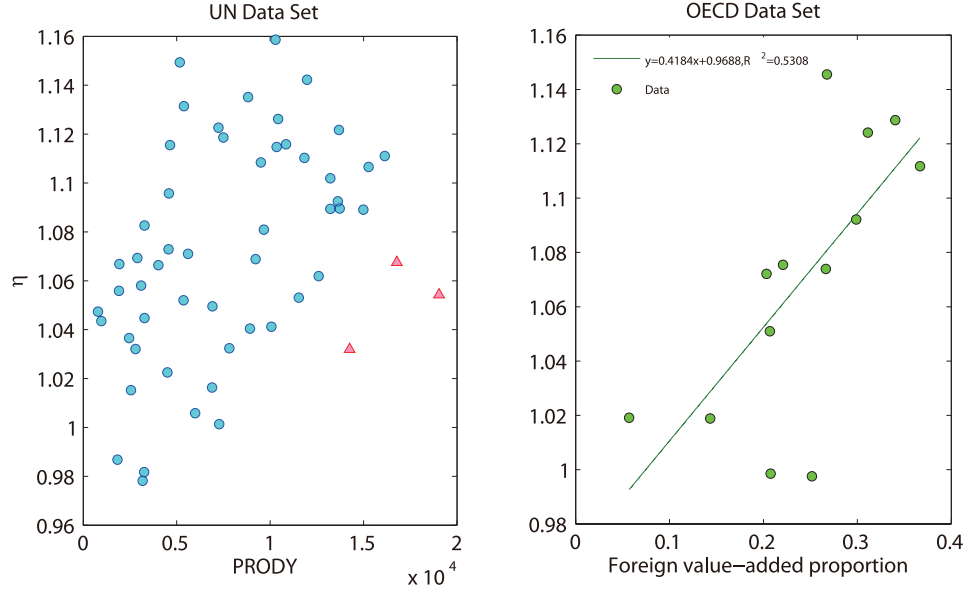
where,  $Y_c$  is the GDP per capita of country  $c$ , and  $RCA(c, p)$  is the comparative advantage of country  $c$  exporting  $p$ . The summation is taken for all of the countries exporting  $p$ .  $RCA(c, p)$  can be calculated as  $RCA(c, p) = \frac{E(c, p) / \sum_p E(c, p)}{\sum_c (E(c, p) / \sum_p E(c, p))}$ , where  $E(c, p)$  is the total export value of  $c$  on  $p$ .

Figure 5 shows the relationship between the exponent  $\eta$  and PRODY of each product using the 2-digit classification of the UN dataset. The correlation coefficient of these two variables is 0.37, and it can be improved to 0.44 if the three outliers (triangles) in Figure 5 are omitted.

For the OECD dataset, the domestic and foreign value added for each product-country combinations are available (see SI). This enables us to correlate exponents with the average foreign value-added ratio of each product. Here, the proportion of foreign value added is the ratio between the total value added and gross export for all of countries exporting this product [1]. The relationship between  $\eta$  and foreign value-added proportion is shown in the right plot of Figure 5. There is a clear positive correlation between them, and the correlation coefficient is 0.692.

Consequently, we conclude that the allometric exponent  $\eta$  of each trade flow network can characterize the complexity and value-added proportion of given product. When a product needs more complex production processes, more countries must be involved to form a long value chain, so that more value is added on the product. All of these properties must be reflected in the flow structure of the product trade network. That is the reason why the allometric exponent  $\eta$  can be distinct for different products.





**Figure 5.** The relationship between  $\eta$  and PRODY of each 2-digit classification(left) in the UN dataset and  $\eta$  versus mean proportion of foreign value added for products in the OECD dataset.

## Discussion

### Country Impacts

In addition to the structural properties of the entire network, the node positions in the global value chain are also of importance and interests. In our study,  $C_i$ , the total impact of country  $i$  toward the entire network, can be viewed as a vertex centrality indicator because it measures the degree of the entire network is influenced if node  $i$  is removed. This understanding is in accordance with the standard HEM (Hypothetical Extraction Method) [28, 29] in input-output analysis once the trade flow networks are understood as an input-output matrixes.

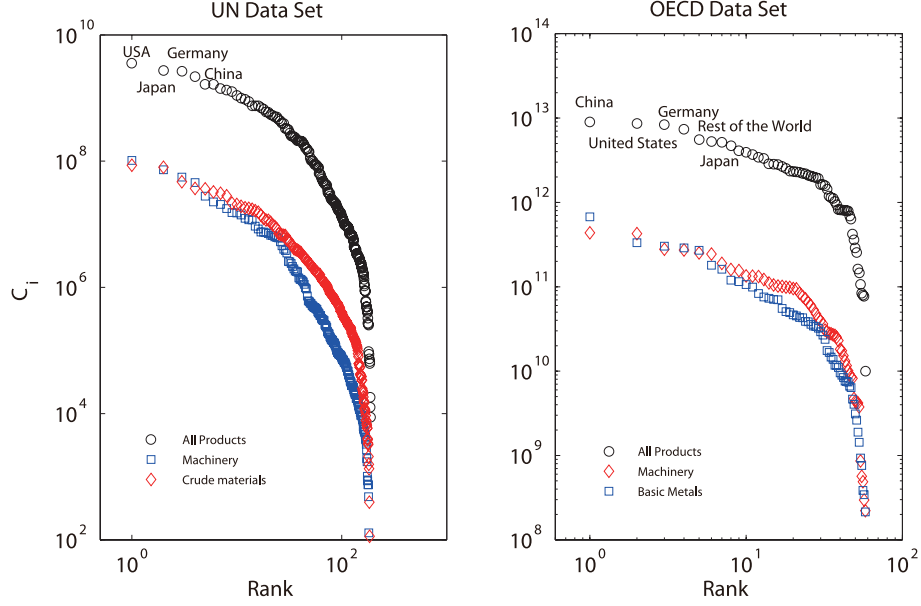
Figure 6 shows the distributions of  $C_i$  for trade networks of all products and several selected products both in the UN and OECD datasets. In addition, the top 10 countries are listed in the Supplementary Information.

### Centrality and Inequality

In our previous works exploring allometric scaling on ecological flow networks [25], the exponent  $\eta$  is explained as the degree of centrality, i.e., whether several large nodes dominate and have a disproportional impact on the entire network. This explanation can also be extended to this study. The networks with higher  $\eta$ s are more centralized. Thus, a few large countries can impact the entire network, in which the impact's degrees  $C_i$  are disproportional to their direct trade flow  $T_i$ .

For example, we have three flow networks with the same  $T_i = \{1, 2, 3, 4, 5\}$  but different  $\eta = \{1, 1/2, 2\}$ . Then, their  $C_i$ s are  $C_i^{(1)} = \{1, 2, 3, 4, 5\}$  for  $\eta = 1$ ,  $C_i^{(2)} = \{1, 1.4, 1.7, 2, 2.2\}$  for  $\eta = 1/2$  and  $C_i^{(3)} = \{1, 4, 9, 16, 25\}$  for  $\eta = 2$ , respectively. As a result, the largest country (the node with the largest  $T_i$ ) dominates  $5/(1+2+3+4+5) \approx 33\%$ ,  $2.2/(1+1.4+1.7+2+2.2) \approx 27\%$ , and  $25/(1+4+9+16+25) \approx 45\%$  impacts of the entire networks, respectively. Therefore, the third network is much more centralized than





**Figure 6.**  $C_i$  Distributions of Both Datasets. The unit of  $C_i$  is the US dollar

the second network.

However, the inequality of exporting products is mainly from the heterogeneity of the resource distribution but not the network effect, which is characterized by  $\eta$ . For example, petroleum export is heterogenous because of the unevenness of fossil fuel resource distribution geographically. Therefore, new indicator is needed.

We use the GINI coefficient of  $C_i$  distribution to characterize the overall inequality of the flow network structure. The  $C_i$  distribution can account for both inequality origins: natural resource distribution and network effects. First, it is obvious that the natural inequality of resource distribution can be reflected by the  $T_i$  distribution. Suppose  $T_i$  follows a Zipf law,  $T_i(r) \sim r^{-\alpha}$ , where,  $\alpha$  is the Zipf exponent, and  $r$  is the rank order of  $i$ . We know that there is a power law relationship between  $T_i$  and  $C_i$  according to Equation 1. Thus,  $C_i$  also follows the Zipf law:  $C_i(r) \sim r^{-\beta} = r^{-\alpha\eta}$ , where  $\beta = \alpha\eta$  is its exponent. Therefore, the distribution of  $C_i$  ( $\beta$ ) contains both types of information: natural heterogeneities ( $\alpha$ ) and network effect( $\eta$ ).

Although  $C_i$  does not follow the Zipf distribution in our empirical data (shown in Figure 6), the previous conclusion that the distribution of  $C_i$  contains both types of information, is still correct. Typically, the GINI coefficient (bounded by  $[0,1]$ ) can be used to characterize the inequality of a variable no matter what type of distribution it follows.

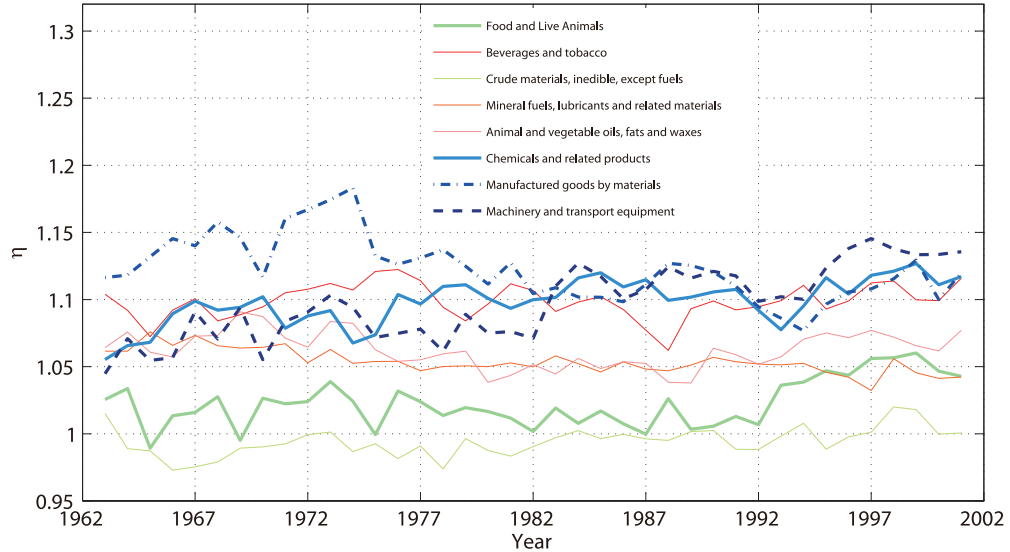
In the last column of Table 1, we show the GINI coefficients of all 1-digit product categories. Most products have similar rank order by GINI as the order by  $\eta$ . However, the order of manufactured goods (Code 6) falls down from No. 2 (by  $\eta$ ) to No. 5 (by GINI coefficient), and the order of food and live animals falls from No. 6 to the bottom. These differences indicate that these two types of products are not so unequal as predicted by the exponent  $\eta$  because the average trading volumes ( $T_i$ ) distribute evenly among countries although their trading networks are more centralized. In the last column of Table 2, the GINI coefficients of all industries of the OECD dataset are shown. There is a large deviation of the order by  $\eta$  from the GINI coefficients. Some industries, such as mining and textiles, have high ranks of GINI coefficients but low ranks of  $\eta$ . That indicates these industries are resource monopolized. Basic metals

and chemicals have high ranks of  $\eta$  but low ranks of GINI coefficients which means the trade networks of these products are centralized.

Another interesting finding is the exponent of the integral trade network that consists of all trading products is 1.02 (It is 0.94 in the OECD dataset). This value is less than the mean exponent when averaging all individual products. This is also found for the GINI coefficients. These findings imply international trade of all products in general becomes much more decentralized than each single product's trade. Therefore, trade in diverse types of products can make the world flatter. Though we still do not know to what degree this conclusion could be true. This will be left for further investigation.

## Exponents in Different Years

The UN dataset records the international trade data historically from year 1962 to 2000. This enables us to study the dynamics of exponents. In Figure 7, we show how these exponents change with time.



**Figure 7.** Allometric exponents  $\eta$ s of 1-digit classification products change with time

Most exponents are almost stable. However, machinery, transport equipment and manufactured material goods have large changes. The latter has very large exponents before 1982, but the former climbs to the top 1 after approximately 1982. Note that some cross-border companies emerged in and around the 1980s. Therefore, machinery and transport equipment products, which depend on vertical labor division but not material, has the largest exponent, whereas the manufactured goods, which are more independent of global cooperation, change in an opposite direction. Thus, the dynamics of the exponents may reflect the globalization process.

## Methods

### Flow Network Model

A flow network model can be built for each product category. Nodes on the network are countries. Directed edges are trading relationships between countries and weights on edges are trading flows measured by the unified money units (the US dollar in our datasets).

If there are in total  $N$  countries participating in trade of the focus product  $p$ , then a flow network can be represented by an  $N \times N$  flux matrix  $F^p$ , in which the element  $f_{ij}^p$  stands for the trade flow of  $p$  from  $i$  to  $j$ . The superscript  $p$  will be omitted to facilitate our expression. All of the variables as well as the trade networks in the following sections are defined for one specific product.

### Trading Volume and Impact

After the construction of the network, we can calculate two important variables for each country. First,  $T_i$  defined as the trading volume of country  $i$ , is the maximum value of either import or export,

$$T_i = \max\left(\sum_{j=1}^N f_{ji}, \sum_{j=1}^N f_{ij}\right). \quad (3)$$

This value measures the amount of product  $p$  flows through country  $i$ , and  $T_i$  reflects the flow capacity that country  $i$  can import or export  $p$ .

However, this indicator can only characterize the direct importance of country  $i$  without the network effect consideration. As trade networks are understood as production chains in our study, one country's export not only influences the direct neighbors but also indirect neighbors along the production chain.

Therefore, we use another variable  $C_i$  to indicate the impact of country  $i$  on the entire network.  $C_i$  is defined as the total reduction of trade volume of all countries if  $i$  is deleted in the network. We will introduce its calculation.

Before  $C_i$  is defined, we should introduce another important matrix  $M$ . It is analogous to the technical coefficient matrix in input-output theory:

$$m_{ij} = \frac{f_{ij}}{T_i}. \quad (4)$$

Thus,  $m_{ij}$  measures the ratio of the export from  $i$  to  $j$  to the total trade volume of  $i$ . Then, the following identity can be derived:

$$T = MT + S, \quad (5)$$

where,  $T = (T_1, T_2, \dots, T_N)^T$ ,  $S = (S_1, S_2, \dots, S_N)^T$ . In addition,

$$S_i = T_i - \sum_{j=1}^N f_{ji} \quad (6)$$

can be viewed as the total domestic value added from  $i$  (see the discussion in SI). Then, we can obtain an important identity from Equation 4:

$$T = (I - M)^{-1} \cdot S, \quad (7)$$

where  $I$  is the identity matrix. Now, suppose node  $i$  is deleted in the network. Then the  $i$ th column in  $M$ , and also  $S_i$  will be set to 0 according to the HEM method [28, 29]. Suppose  $M$  turns into  $M'$  and  $S$  turns into  $S'$ . Then, the new total trade volume vector can be computed if we believe the identity Equation 7 also holds for  $M', S'$  and  $T'$ :

$$T' = (I - M')^{-1} \cdot S'. \quad (8)$$

Then, the total amount of trade volume reduction in the entire network is defined as  $C_i$ ,

$$C_i = (1, 1, \dots, 1) \cdot (T - T'). \quad (9)$$

To ease our calculation, we always use the following equation

$$C_i = \sum_{k=1}^N \sum_{j=1}^N S_j \frac{u_{ji} u_{ik}}{u_{ii}}, \quad (10)$$

where  $U = (I - M)^{-1}$ . It can be proved that Equation 10 equals Equation 9 (see SI).

## Network Allometry

Allometric scaling is a universal pattern of transportation networks including rivers and vascular networks. Previous studies on network allometry can only be applied to directed trees, in which  $T_i$  is the total number of nodes in the sub-tree rooted from  $i$  and  $C_i$  is the summation of all  $T_i$ s in the sub-tree rooted from  $i$  [20, 21].

The allometric exponents for trees are bounded between 1 and 2. The minimum exponent can be obtained by a star-like network, in which all links are from the root to other nodes, whereas the maximum exponent is obtained by a chain as shown in Figure 1. These two special trees stand for two extremes. The star-like tree is flat because every node except the root is equivalent. However, the chain-like tree is hierarchical because the nodes at the upper level dominate the other nodes at the lower level.

The allometric exponents can be computed for any tree. If the exponent is smaller, then it is more star-like. Otherwise, it is more chain-like.

Our previous works extended these studies to general flow networks [22, 25]. By incorporating the energy flow analysis technique, we can assign an exponent for any flow network without edge cutting. By calculating the  $T_i$  and  $C_i$  for each node  $i$  according to the method introduced in the previous subsection, we can obtain the allometric exponent by estimating the slope of  $\log T_i$  versus  $\log C_i$ .

The exponent  $\eta$  can be viewed as the level of hierarchicality of the flow structure because the relative speed of  $C_i$ , can increase faster than  $T_i$  in a network with a large exponent. As the extension of the allometry of spanning trees, the flow network is more like a chain if its exponent is large. Therefore, some long flow chains can be revealed in these networks.

We distinguish networks as hierarchical ( $\eta > 1$ ), neutral ( $\eta \approx 1$ ) and flat ( $\eta < 1$ ) using the exponent.

## Supporting Information Legends

Table S1. The dataset form in UN dataset. Table S2. The trade data in OECD dataset. Table S3. The value added data in OECD dataset. Table S4. The result of computed according to (4) and (5). Table S5. Exponents of Leamer Classification Standard. Table S6. The top ten  $C_i$  of different products in UN dataset. Table S7. Top ten countries of different industries in the OECD Dataset. Figure S1. Balanced value flow of one country. Figure S3. The relationship between and the mean proportion of foreign value added

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